We reviewed Hamming codes, the dimensions of orthogonal complements, and constructed the generating matrix for the code from the HW. Those who turned in the HW on time get a 10pt bonus.

**You have until Monday noon to turn in the HW to Arvana.**

Monday you will work on the distance decoding problems, Wednesday, October 12, will be our first exam.

Then we returned back to the ideas about normalizers. We constructed several normalizers in the group $D_4$ and stated several informal lemmas. Informal is nice, but then I make mistakes. So, first of all, I was wrong about $N_G(G)$, as you probably already know. Namely:

For every group $G$ (abelian or not), $N_G(G) = G$. The argument is very straightforward, and you should have called me on that – you do not need to put up with my mistakes! Anyway: for every $g \in G$ and every $s \in G$, obviously $g^{-1}g \in G$. Hence, every $g \in G$ belongs to $N_G(G)$, i.e., $N_G(G) = G$.

I do not know what I was thinking, obviously of $C_G(G)$ which is indeed a proper subgroup of $G$ for non-abelian $G$.

I asked you to determine what does $S_1 \subseteq S_2$ imply for $N_G(S_1)$ and $N_G(S_2)$. I hope that you tried it, and know by now that nothing can be said in general for these. It can go both ways:

Let $G = D_4$. Let $S_1 = \{r\}$ and $S_2 = \{1, r, r^2, r^3\}$. Then $N_G(S_2) = G$ (verify!), while $N_G(S_1)$ does not contain $s$ because $srs = r^3 \not\in S_1$, as all powers of $r$ obviously do belong to $N_G(S_1)$ (verify!), ”number magic” tells me that $N_G(S_1) = \{1, r, r^2, r^3\}$. I.e., it is possible for a smaller set to have a smaller normalizer.

However, it can go the other way as well: Let $G = D_4$ and this time take $S_1 = \{r^2\}$ and $S_2 = \{r, r^2\}$. Then $N_G(S_1) = G$ (verify!) and $N_G(S_2) = \{1, r, r^2, r^3\}$. I.e., a smaller set can also have a larger normalizer. *No general statement is possible!*

By the end of the class we started to discuss centralizers: if $\emptyset \neq S \subseteq G$, then

$$C_G(S) = \{g \in G \mid gsg^{-1} = s, \forall s \in S\}$$

Again, we did some examples and proofs, and said that $C_G(G)$ has a special name – the center $Z(G)$, of $G$, and it is the set of all elements in $G$ that commute with any other element.

Relevant material can be found in Dummit and Foote, Section 2.2.

**Suggested exercises:**
Section 2.2: p53 / 2, 3, 4, 6, 10