We continued discussing the automorphism groups of graphs. We computed the full automorphism group of a graph on three vertices with two edges. Then we observed and proved the fact that the automorphism group of a complementary graph is the same as the auto group of the original graph. We used this observation to classify the auto groups of all 3-vertex graphs.

Open question to work on: Find a graph with a trivial auto gp.

We went through the complete list of all 4-vertex graphs, and argued its completeness as well as discusses the necessary conditions for two graphs to be isomorphic (equal number of vertices, edges, equal vertex degree distributions). We also argued that none of those auto gps is trivial.

Finally, we observed that the complete graph and its complement have for their auto gps the full symmetric group. Discussed the full symmetric groups, and introduced the cycle notation for permutations.

At the end we recalled the definition of the order of a group element, and the formula for the order of an element given by its cycle decomposition.

The list of groups we need to get comfortable with includes:

- **cyclic groups** \((\mathbb{Z}_n, +)\)
- **multiplicative groups of modulo arithmetic** \((\mathbb{Z}_n^*, \cdot)\) of all invertible elements under multiplication modulo \(n\).
  
  Note: invertible are all the elements that are relatively prime to \(n\).
- **dihedral groups** \(D_n\) of all structure preserving automorphisms of an \(n\)-gon
- **symmetric groups** \(S_n\) of all permutations of an \(n\)-element set
- **matrix or general linear groups** \(GL_n(F)\) of all invertible \(n \times n\) matrices over a field \(F\) under multiplication of matrices; with \(F\) being one of the fields \(Q, R, \mathbb{Z}_p\)
- **quaternion group** \(Q_8 = \{1, -1, i, -i, j, -j, k, -k\}\)

For basic information on these groups, consult any text on group theory, e.g., Dummit and Foot.

Possible practise exercises include:
Section 1.1. : p21 / 1, 2, 5, 9, 10, 12, 16, 17, 20, 22, 25, 28, 35
Section 1.2. : p27 / 1, 3, 4, 5, 7, 9, 10, 15, 18
Section 1.3. : p33 / 1, 2, 4, 5, 6, 7, 8, 9, 11, 14, 15, 18, 19
Section 1.4. : p35 / 1, 2, 3, 4, 6, 10
Section 1.5. : p36 / 1, 2, 3