We continued discussing the automorphism groups of graphs. We considered two examples of \textbf{graphs with trivial automorphism group}.

Then we discussed \textbf{matrix groups}, with specific attention to matrix groups over finite fields. We said that we would mostly focus on matrix groups over \((\mathbb{Z}_p, +, \cdot)\). We counted and developed a formula for the number of elements in \(GL_n(\mathbb{Z}_p)\).

Afterwards, we defined and discussed the group \(Q_8\) of \textbf{quaternions}. You should be able to complete the multiplication table for \(Q_8\).

Finally, we started to discuss \textbf{group isomorphisms}, i.e., bijections between groups that preserve the structure of the group.

You should be able to prove the following statements:

If \(\varphi : G \to H\) is a group automorphism, then:

\begin{itemize}
  \item \(\varphi(1_G) = 1_H\)
  \item \(|g| = |\varphi(g)|\), for all \(g \in G\) (where \(|a|\) denotes the order of the element \(a\))
  \item \(\varphi(g^{-1}) = (\varphi(g))^{-1}\)
\end{itemize}

Possible practice exercises include:

- Section 1.1. : p21 / 1, 2, 5, 9, 10, 12, 16, 17, 20, 22, 25, 28, 35
- Section 1.2. : p27 / 1, 3, 4, 5, 7, 9, 10, 15, 18
- Section 1.3. : p33 / 1, 2, 4, 5, 6, 7, 8, 9, 11, 14, 15, 18, 19
- Section 1.4. : p35 / 1, 2, 3, 4, 6, 10
- Section 1.5. : p36 / 1, 2, 3