Some Observations on Subset Sum Representations

Moulton and Develin have investigated the notion of representing various nonempty sets $S \subseteq \mathbb{Z}^+$, $|S| = m < \infty$, as subset sums of smaller sets. Specifically, the rank of a given set $S$ is the smallest positive integer $rk(S) \leq m$ such that $S$ can be represented by an integer set of size $rk(S)$. For example, the set $S = \{1, 2, 4, 8, 16\}$ can be represented by the elements of the set $T = \{-5, 1, 7, 9\}$, as $2 = 7 + (-5)$, $4 = 9 + (-5)$, $8 = 1 + 7$ and $16 = 7 + 9$.

As it can be proved that $S$ cannot be represented by a set of size less than 4, $rk(S) = 4$. As $rk(S) < |S|$, $S$ is said to be dependent, where the notion of dependence can be understood from a linear algebraic standpoint. If $rk(S) = |S|$, then $S$ is independent.

For an infinite set $S \subseteq \mathbb{Z}^+$ whose first $k$ elements constitute the subset $S_k$, the limiting rank of $S$, namely $\lim_{k \to \infty} \left[ \frac{rk(S_k)}{k} \right]$ (provided said limit exists), is also a matter of particular interest. In this talk, I will present new results concerning sets of the form $S = \{1, 2^m, 3^m, \ldots\}$ for $m \in \mathbb{Z}^+$. In one direction, we consider what happens when the cardinality of $S$ is fixed, say $|S| = k$. Given a positive integer $k$, we ask for the smallest $M$ such that $\{1, 2^m, 3^m, \ldots, k^m\}$ is independent for all $m \geq M$, and provide some answers. We will then use a result of Sprague to prove that any nondecreasing positive integer sequence $a = \{a_1, a_2, \ldots\}$ that grows polynomially, and in particular the set $\{1, 2^m, 3^m, \ldots\}$ for fixed exponent $m$, has limiting rank zero.

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