Math 380 Homework Assignment #2.
Due Tuesday, February 11., in the class.
Read the problems carefully and write the whole solution. It never hurts to write more
than less. You are not allowed to discuss the problems with anybody but the instructor.

1. (5pts) Use the truth table to determine whether the proposition \((P \land Q) \lor (P \land \neg R)\)
is a negation of the proposition \((\neg P \lor \neg Q) \land (\neg P \lor \neg R)\).

2. (5pts) Is it possible for two formal propositions one of which contains just the atomic
proposition \(P\) and the other one contains just \(\neg P\) (and any connectives you desire)
to be logically equivalent? Justify your answer.

3. (5pts) Find a tautology (i.e., a statement that is true for all choices of truth values
for its atomic propositions) that contains at least four different atomic propositions.
Use the truth table to prove that your proposition really is a tautology.

4. (10pts) Consider the formula : \((P \land Q) \lor (P \lor \neg Q)\).
a) (5pts) Find two English sentences \(S_1\) and \(S_2\) that, if substituted for \(P\) and \(Q\) respectively,
will make the above formula a true proposition.
b) (5pts) Find two English sentences \(S_1\) and \(S_2\) that, if substituted for \(P\) and \(Q\) respectively,
will make the above formula a false proposition.

Please use sentences for which I can easily determine whether they are true or not. For
instance, I cannot determine very easily the truth value of the statement “I am mar-
ried and have two children” while I can see that the sentence “George Washington
is the current US president” is false.

5. (5pts) Find a formal proposition with three atomic propositions that is true exactly
for five of the eight possible truth assignments.

5. (5pts) Find the negation of the proposition :

\[(P \Rightarrow \neg Q) \lor (Q \Rightarrow P)\]

6. (5pts) Is there an open sentence \(P(x)\) such that both \((\forall x)(P(x))\) and \((\exists x)(P(x))\) are
false with respect to the same universe? If so, give an example. If your answer is
"no", give reasons for your decision.

7. (5pts) Is the following proposition true for all reals?
\[(\forall x \in R)(\forall y \in R)(\exists ! z \in R)(xz = y)\]

8. (5pts) Find a theorem in calculus that is of the form

\[P \Rightarrow (Q \lor S)\]